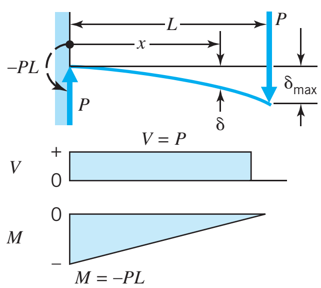
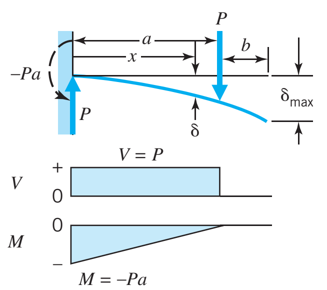
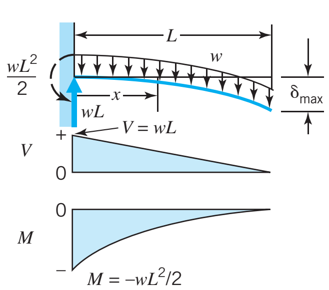
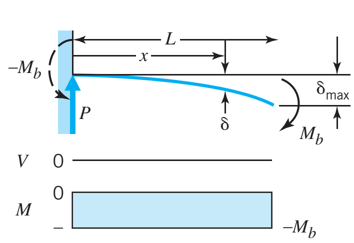
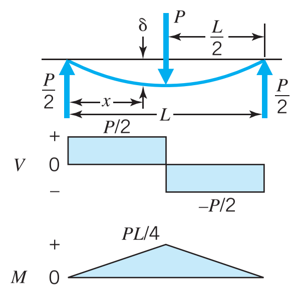
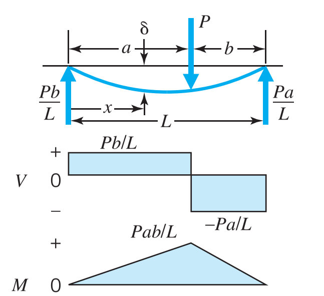
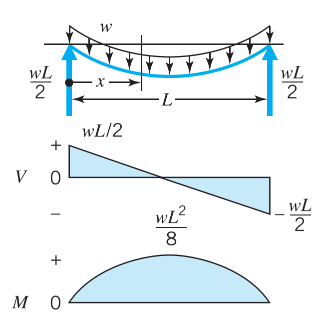
APPENDIX D

Shear, Moment, and Deflection Equations for Beams

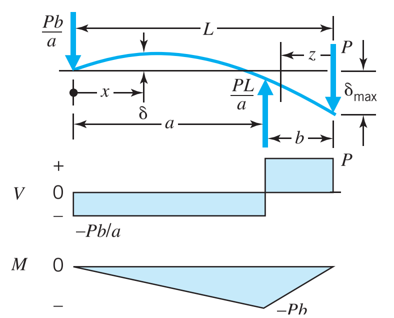
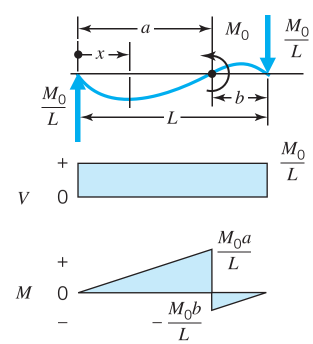
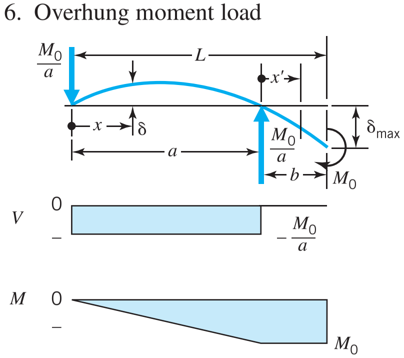
Appendix D-1 *Shear, Moment, and Deflection Equations for Cantilever Beams*

	Slope at Free End	Maximum Deflection	Deflection δ at Any Point x
1. Concentrated load at end  	$\theta = \dfrac{PL^2}{2EI}$	$\delta_{max} = \dfrac{PL^3}{3EI}$	$\delta = \dfrac{Px^2}{6EI}(3L - x)$
2. Concentrated load at any point  	$\theta = \dfrac{Pa^2}{2EI}$	$\delta_{max} = \dfrac{Pa^2}{6EI}(3L - a)$	For $0 \le x \le a$: $\delta = \dfrac{Px^2}{6EI}(3a - x)$ For $a \le x \le L$: $\delta = \dfrac{Pa^2}{6EI}(3x - a)$
3. Uniform load  	$\theta = \dfrac{wL^3}{6EI}$	$\delta_{max} = \dfrac{wL^4}{8EI}$	$\delta = \dfrac{wx^2}{24EI}(x^2 + 6L^2 - 4Lx)$
4. Moment load at free end  	$\theta = \dfrac{M_bL}{EI}$	$\delta_{max} = \dfrac{M_bL^2}{2EI}$	$\delta = \dfrac{M_bx^2}{2EI}$

Appendix D-2 *Shear, Moment, and Deflection Equations for Simply Supported Beams*

	Slope at Ends, θ	Maximum Deflection, δ_{max}	Deflection δ at Any Point x
1. Concentrated center load 	$\frac{PL^2}{16EI}$	At center: $\frac{PL^3}{48EI}$	For $0 \le x \le L/2$: $\frac{Px}{12EI}\left(\frac{3L^2}{4} - x^2\right)$
2. Concentrated load at any point 	At left end: $\frac{Pb(L^2 - b^2)}{6LEI}$	At $x = \sqrt{\frac{L^2 - b^2}{3}}$: $\frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}LEI}$	For $0 \le x \le a$: $\frac{Pbx}{6LEI}(L^2 - x^2 - b^2)$
3. Uniform load 	$\frac{wL^3}{24EI}$	$\frac{5wL^4}{384EI}$	$\frac{wx}{24EI}(L^3 - 2Lx^2 + x^3)$

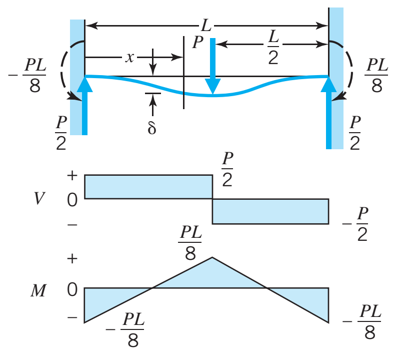
Appendix D-2 *(continued)*

	Slope at Ends, θ	Maximum Deflection, δ_{max}	Deflection δ at Any Point x
4. Overhung load 	At left support: $\frac{Pab}{6EI}$ At right support: $\frac{Pab}{3EI}$ At load: $\frac{Pb}{6EI}(2L + b)$	$\delta_{max} = \frac{Pb^2L}{3EI}$	For $0 \le x \le a$: $\frac{Pbx}{6aEI}(x^2 - a^2)$ For $0 \le z \le b$: $\frac{P}{6EI}[z^3 - b(2L + b)z + 2b^2L]$
5. Moment load between support 	At left support: $\frac{-M_0}{6EIL}(2L^2 - 6aL + 3a^2)$ At load: $\frac{M_0}{EI}\left(\frac{L}{3} + \frac{a^2}{L} - a\right)$ At right support: $\frac{M_0}{6EIL}(L^2 - 3a^2)$	At load: $\frac{M_0a}{3EIL}(2a^2 - 3aL + L^2)$	For $0 \le x \le a$: $\frac{M_0x}{6EIL}(x^2 + 3a^2 - 6aL + 2L^2)$
6. Overhung moment load 	At left support: $\frac{M_0a}{6EI}$ At right support: $\frac{M_0a}{3EI}$ At load: $\frac{M_0(a + 3b)}{3EI}$	$\delta_{max} = \frac{M_0b}{6EI}(2L + b)$	For $0 \le x \le a$: $-\frac{M_0x}{6aEI}(a^2 - x^2)$ For $0 \le x' \le b$: $\frac{M_0}{6EI}(2ax' + 3x'^2)$

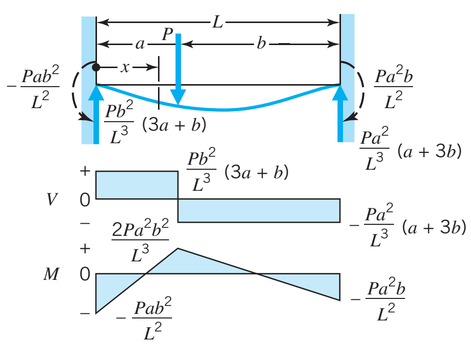
Appendix D-3 *Shear, Moment, and Deflection Equations for Beams with Fixed Ends*

	Deflection δ	Deflection δ at Any Point x
1. Concentrated center load	At center: $\delta_{max} = \dfrac{PL^3}{192EI}$	For $0 \le x \le L/2$: $\delta = \dfrac{Px^2}{48EI}(3L - 4x)$
2. Concentrated load at any point	At load: $\delta = \dfrac{Pb^3a^3}{3EIL^3}$	For $0 \le x \le a$: $\delta = \dfrac{Pb^2x^2}{6EIL^3}[3aL - (3a + b)x]$
3. Uniform load	At center: $\delta_{max} = \dfrac{wL^4}{384EI}$	For $0 \le x \le L$: $\delta = \dfrac{wx^2}{24EI}(L - x)^2$

1. Concentrated center load



2. Concentrated load at any point



3. Uniform load